Phys 402
Fall 2022
Homework 9
Due Wednesday, November 9, 2022 @ 10 AM

Mid-term EXAM 2 will be on Wednesday 16 November (10:00 AM to 11:50 AM), covering everything up to and including HW 9

1. Griffiths, 3rd Edition, Problem 9.1 **[WKB for "shelf" potential in the infinite square well]**

2. Griffiths, 3rd Edition, Problem 9.2 **[WKB as a perturbation expansion in ħ]**

3. Griffiths, 3rd Edition, Problem 9.3 **[WKB tunneling transmission probability]**

4. Griffiths, 3rd Edition, Problem 9.5 **[Zener Tunelling]** {*Hint: The electron has to tunnel through a triangular barrier of height E_g (the energy gap) on one side*}

5. Griffiths, 3rd Edition, Problem 8.1 **[Variational estimate of ground state energy in linear and quartic potentials]**

6. Griffiths, 3rd Edition, Problem 8.19 **[Variational estimate of ground state energy of Hydrogen]**

Extra Credit 9:

The Schrödinger equation for the Macroscopic Quantum Wavefunction Ψ(**r**,t) for a superconductor is $i\hbar\frac{\partial \Psi}{\partial t} = \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} - q^*\vec{A}\right)^2\Psi + q^*\phi\Psi$, where $\vec{A}$ is the vector potential, ϕ is the scalar potential, m^* and q^* are the effective mass and charge of a Cooper pair. The macroscopic quantum wavefunction is interpreted as $\Psi(\vec{r},t) = \sqrt{n^*(r,t)}\,e^{i\theta(\vec{r},t)}$, $n^*(\vec{r},t)$ is the local number density and $\theta(\vec{r},t)$ is the space and time-dependent phase.

a) Under the assumption that the number density $n^*(\vec{r},t) = |\Psi(\vec{r},t)|^2$ is constant in space and time, derive the energy-phase relationship:

$$-\hbar\frac{\partial \theta}{\partial t} = \frac{1}{2n^*}\Lambda J_s^2 + q^*\phi$$

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from the real part of the macroscopic quantum Schrödinger equation. Interpret this equation physically. Here the supercurrent density $\vec{J}_s = \frac{1}{\Lambda}(\frac{\hbar}{q^*}\vec{\nabla}\theta - \vec{A})$ and $\Lambda = \frac{m^*}{n^*(q^*)^2}$.

b) Now assume that $n^*(\vec{r},t)$ is NOT constant in either space or time. Show that the imaginary part of the macroscopic Schrödinger equation yields:

$$\frac{\partial n^*}{\partial t} = -\vec{\nabla} \bullet (n^* \vec{v}_s)$$

Interpret this result physically (it may help to multiply both sides by q*). Note that the superfluid velocity is given by $\vec{v}_s = \frac{\hbar}{m^*}\vec{\nabla}\theta - \frac{q^*}{m^*}\vec{A}$